Three FM Methods for Measuring Tracking Angles of Phono Pickups*

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Three FM-distortion methods for measuring the vertical tracking angles of phono pickups are described and compared. Data from ten pickups verify that the three FM methods give results consistent with each other to within the calibration tolerance of the test records and measuring equipment, which is estimated to be ±5%. In addition, the vertical-angle values obtained by the FM methods agree on the average with angle values obtained by an optical measurement technique. One of the FM methods is particularly suited to production-line testing for quality control because precise measurements of the vertical angle can be obtained in "real time" while a single band on the test record is being played.

0. INTRODUCTION

The vertical tracking angle is an important mechanical parameter of a phono pickup because it influences the linearity of sound reproduction. It is defined as the angle between two straight lines: one normal to the plane of the phonograph record and the other in the direction along which the stylus vibrates on the average while reproducing a purely vertical modulation on the record. To avoid vertical tracking distortion, this angle must equal the effective vertical angle with which the record grooves were recorded.

For an ideal pickup with a rigid stylus shank, the vertical tracking angle θ_P can be visualized as shown in Fig. 1, which illustrates a stylus assembly in relation to the surface of the recording. The figure suggests (in an exaggerated form) that the dynamic pivot point may not be located along the stylus shank when an audio signal is reproduced.

In 1962 Madsen [1] presented data showing that the vertical angles of commercial pickups varied over a substantial range. This was confirmed by research at CBS Laboratories where Bauer [2], [3] investigated different methods for measuring both pickup angles and the effective vertical angles of cutting heads. He discovered that the effective recording angles did not match the geometric angles computed from a kinematic analysis of the cutting heads then used. He also pointed out that this discrepancy, which amounted to more than 20 degrees, could be explained as the combined effects of elastic deformation of the cutting stylus assembly and elastic springback of the lacquer (cellulose nitrate) in the recording blank.

Comparisons of different methods for measuring vertical angles were published by Halter and Woodward [4]; they concluded that measurements of the frequency modulation (FM) caused by tracking-angle error offered the most accurate means of determining the vertical angles of pickups because the data were obtained under actual playing conditions and were not biased by amplitude-modulation distortions caused by other nonlinearities in the recording–reproducing chain. This conclusion was also reached by Cooper [5] in his detailed analysis of different measurement techniques.

At the present time there are two FM techniques for measuring vertical angles with commercially available test records. The first is a method described by Woodward [6], which uses the RCA 12-5-78 test record, while the second uses the CBS STR-112 test record as described by White and Gust [7], [8].

In this paper we present data on the vertical tracking angles of ten pickups, which were measured by three different FM techniques (the two just mentioned plus a third that was recently developed at CBS) and also by a fourth

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optical method in which no test record is used. The three FM techniques yielded tracking-angle values that are precise and in good agreement with each other; the average differences in tracking angles between these methods are comparable to the calibration tolerances of the test records and measuring equipment (±5%). In addition, there was no statistically significant average difference between the vertical angles determined by optical and FM methods.

1. DESCRIPTION OF TEST METHODS

1.1 General

For convenience in this paper, we have given the following names to the three FM test methods. The first, which uses the RCA 12-5-78 test record, is called the "total-sine-wave method." The second, which uses the CBS STR-112 test record, is called (for reasons that will be explained) the "real-sine-wave method." And the third, which uses a special test record that is not commercially available, is called the "total-square-wave method."

1.2 Total-Sine-Wave Method

The total-sine-wave method uses the RCA 12-5-78 test record [6] in which a large-displacement 400 Hz tone is recorded vertically together with a small-displacement 4-kHz tone. The 4-kHz tone acts as a carrier that is frequency modulated by the 400-Hz tone during playback when there is a tracking-angle error, that is, when the angle of the pickup differs from the effective angle with which the record was cut. (Tracing errors can also cause FM distortion of the 4-kHz carrier and so can other departures from ideal behavior in the pickup and the stylus–groove interaction. However, when the tracking-angle error is large enough, then the greater part of the total FM distortion is tracking distortion.)

The total-sine-wave method consists in measuring the total 400-Hz component of frequency modulation of the 4-kHz carrier. The experimental setup used for this method is shown by the block diagram in Fig. 2. The amplified vertical output of the pickup was fed to the 2-kHz to 5-kHz bandpass filter, which reduced the level of the 400-Hz recorded signal 56 dB so as not to disturb the FM demodulation that was performed by the discriminator (General Radio type 1142-A). The filters at the output of the discriminator isolated the 400-Hz component of the frequency deviation, and the voltmeter measured voltage E, which was proportional to the peak frequency deviation. Vertical-angle values of high resolution can be computed from measurements of voltage E by the following procedure, which is based on a method first described by Cooper [5].

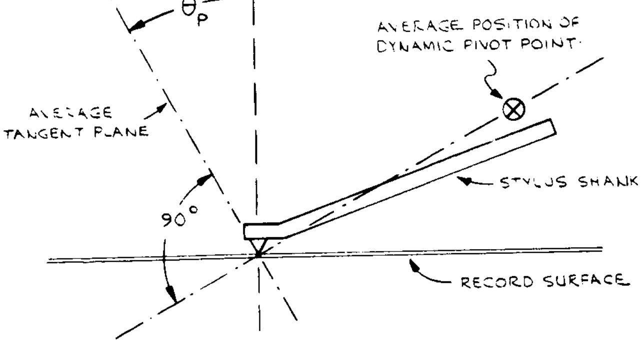

Fig. 1. Visualization of vertical tracking angle θ_P.

The RCA test record has 22 bands, each recorded at a different effective vertical angle. Let the vertical angle of the ith band be denoted by θ_i, and compute the inclination number $y_i = \tan\theta_i$. When the ith band is played with a particular pickup, the voltage E at the output of the discriminator assumes some value, say E_i. Of the 22 numbers E_i, $i = 1 \cdots 22$, one is smaller than the others, say E_m. Let this minimum value be deleted from the data, and let the remaining data be renumbered consecutively, $i = 1 \cdots 21$. Then set $x_i = E_i$ for $i = 1 \cdots m-1$, and $x_i = -E_i$ for $i = m \cdots 21$, and form the ordered pairs $(x_i, y_i) = (\pm E_i, \tan\theta_i)$. (By deleting the smallest datum we avoid a slight statistical bias due to a sign ambiguity in the data.) There are 21 of these ordered pairs generated by playing all bands on the test record with a single pickup. A graphical method for determining the vertical angle of the pickup from these data consists in plotting the pairs (x_i, y_i), $i = 1 \cdots 21$, on linear graph paper with respect to a rectangular x–y coordinate system and then constructing a straight line $y = a + bx$ (a, b being real numbers) that best fits the data. The vertical angle θ_P of the pickup is then given by the formula $\theta_P = \tan^{-1} a$, where a is the y intercept of the line.

A variation of this technique, which is quicker and provides additional statistical information, is to use a programmable calculator to compute the linear regression of y on x, that is, compute the values for a and b that minimize the sum of squared errors:

$$\sum_{i=1}^{i=21} (a + bx_i - y_i)^2$$

In addition, the following statistics are computed to measure the precision with which the regression line fits the data. S_y is the standard error of y on x; S_a the standard error of a; S_b the standard error of b; and S_θ the standard error of the vertical pickup angle θ_P. These statistics are computed from the following formulas [9], where n is the number of ordered pairs (21), and all summations range from $i = 1$ to $i = n$:

$$\begin{aligned}
b &= (\Sigma x_i y_i - \Sigma x_i \Sigma y_i / n)/(\Sigma x_i^2 - (\Sigma x_i)^2/n) \\
a &= \Sigma y_i / n - b \Sigma x_i / n \\
S_y &= [\Sigma y_i^2 - (\Sigma y_i)^2/n - b(\Sigma x_i y_i - \Sigma x_i \Sigma y_i / n)]^{0.5} (n-2)^{-0.5} \\
S_a &= S_y [\Sigma x_i^2 / (n \Sigma x_i^2 - (\Sigma x_i)^2)]^{0.5} \\
S_b &= S_y [\Sigma x_i^2 - (\Sigma x_i)^2/n]^{0.5} \\
S_\theta &\cong S_a \mathrm{d}\theta / \mathrm{d}a \qquad \text{[radian]} \\
&= 57.3 S_a/(1 + a^2) \qquad \text{[degree]} .
\end{aligned}$$

In our measurements with ten pickups and one test record, the standard error S_θ of the vertical angle varied from 0.13

to 0.28 degree, which indicates that the data were precise by engineering standards.

1.3 Real-Sine-Wave Method

The real-sine-wave method of measuring vertical angles was described [7] at the Fall 1977 AES Convention, and a detailed analysis and discussion of it are given in [8]. Therefore, only a description of the experimental procedure and method of data analysis is given here.

The block diagram of the experimental setup is shown in Fig. 3. It is similar to the setup for the total-sine-wave method in Fig. 2, except that either left- or right-channel output of the pickup is measured, and an additional 400-Hz filter and a phasemeter have been added. Three bands on the CBS STR-112 test record are used; namely, group B, which have 400-Hz and 4-kHz tones recorded together in the vertical channel with an effective vertical recording angle of 16.5 degrees and an estimated tolerance of ±1 degree. The purpose of the phasemeter is to make it possible to measure the phase lead of the 400-Hz frequency deviation of the 4-kHz tone with respect to the velocity of the recorded 400-Hz tone. It can be shown [8], [10] that tracking-angle errors produce 400-Hz frequency deviations that are strictly either in phase or out of phase with the recorded 400-Hz velocity. (In contrast, tracing errors produce frequency deviations that are strictly in quadrature). Thus if the 400-Hz voltage waveforms E_1 and E_2 are represented by phasors in the complex plane, and E_1 is defined to be real and positive, then only the real part of E_2 is influenced by the tracking-angle error in a properly calibrated experiment. Because of this, it can be shown [8] that the vertical angle θ_P of the pickup at 400 Hz is given by the following formula when the real FM deviation is caused by tracking-angle errors:

$$\theta_P = \tan^{-1} [0.296 \mp 9.10 \times 10^{-4} R E_P \cos\phi / (k V_{400})]$$

where R is the groove radius in meters; V_{400} the peak velocity in meters per second of the recorded 400-Hz tone; and E_P the peak value of E_2, equalling $k \cdot$ (peak 400-Hz frequency deviation) with k being the frequency-to-voltage gain in volts per hertz of the discriminator and its output filter. ϕ is the phase lead of E_2 with respect to E_1. The minus sign is used for data obtained from the left channel of the pickup output, and the plus sign for right-channel data.

Although only one measurement each of E_P and ϕ from one band on the record are needed to determine the pickup angle θ_P by this method, in our experiments all three bands were played twice, once while acquiring left-channel data and once for right-channel data. In this way, for each pickup six measured values of θ_P were obtained per experiment, from which the mean angle $\bar{\phi}_P$, the standard deviation S, and the standard error of the mean $S_\theta = S/(6)^{0.5}$ were computed. The mean $\bar{\phi}_P$ was taken as the best estimate of the pickup angle, and the standard error S_θ of this mean measured its precision. Data from 17 such experiments (using ten different pickups and two test records) yielded standard errors S_θ that ranged from 0.03 to 0.23 degree, with an average of 0.11 degree, while the corresponding standard deviations ranged from 0.07 to 0.56 degree. These statistics indicate that the data were precise in an engineering sense.

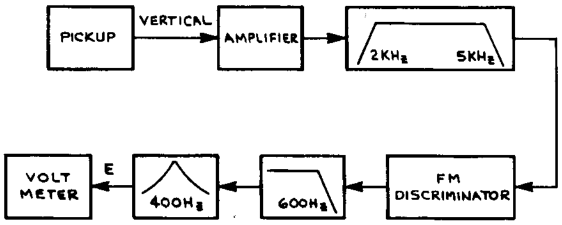

Fig. 2. Block diagram of experimental setup for the total-sine-wave method.

1.4 Total-Square-Wave Method

The last technique for measuring vertical angles, the total-square-wave method, is believed to be new, although it is a natural extension of Cooper's FXM method [5]. The test record required for this method is not available commercially, and so lacquer (cellulose nitrate) disks were cut at the CBS Technology Center specially for these tests. The test signal on this record consists of a 400-Hz velocity square wave recorded vertically, together with a 10-kHz sine wave recorded laterally. In the presence of vertical tracking-angle error, the 10-kHz tone is frequently modulated by the 400-Hz square wave, and it is this FM that is measured. Square waves are used because they are reproduced without tracing distortion (between their level transitions) no matter what the shape or orientation of the stylus or the crosstalk characteristics of the pickup may be.

To understand the experimental procedure, consider Fig. 4 and let us assume that the test record is being played with the switch S in its horizontal position. The frequency-modulated 10-kHz lateral tone then passes through the 8-kHz to 13-kHz filter, where residual components of the 400-Hz vertical square wave (due to crosstalk) are attenuated, and enters the FM discriminator (General Radio type 1142-A). The frequency-deviation signal from the discriminator is then cleaned up with a 10-Hz to 2250-Hz bandpass filter and a 10-kHz notch filter and then passes (through an allpass filter) to the oscilloscope, which displays a ringing square wave with peak-to-peak amplitude A. The magnitude of this square-wave signal is a measure of the vertical tracking-angle error. The gain of

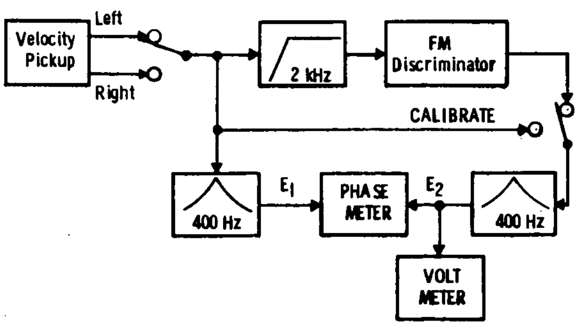

Fig. 3. Block diagram of experimental setup for the real-sine-wave method.

the oscilloscope is adjusted so that amplitude A corresponds to convenient grid markings on the scope face.

This measuring system is calibrated as follows. The filter driving the oscilloscope is now set to 400-Hz bandpass, and switch S is set to the output of the voltage-controlled oscillator (VCO) (Krohn-Hite model 5300-R function generator). The direct current offset driving the VCO is assumed to have been previously adjusted so that the average frequency is 10 kHz (which can be verified with the 10-kHz notch filter), and the gain G is now adjusted so that the 400-Hz sinewave oscillator frequency modulates the 10 kHz by just the amount needed to produce the same peak-to-peak frequency deviation as was previously produced by the test record. This correct setting of gain G is determined by examining the 400-Hz sinewave deviation on the oscilloscope and verifying that the peak-to-peak value is A. Finally, the 400-Hz voltage E needed to produce this deviation in the output of the VCO is measured with a voltmeter. The VCO is assumed to have a known calibration constant k in volts per hertz, such that $E = k \cdot$ (peak-to-peak 400-Hz frequency deviation).

It can be shown from tracking theory [8] that the vertical tracking angle θ_P of the pickup can then be computed from the formula

$$\theta_P = \tan^{-1}[\tan \theta_R \pm E 174.5 \times 10^{-6} R/(kV_{sw} \cos\theta_R)] \quad (1)$$

where the plus sign is to be used if the pickup angle θ_P is greater than the effective recording angle θ_R, while the minus sign is to be used if $\theta_P < \theta_R$. The other symbols are defined as follows: R is the groove radius in meters, and V_{SW} the peak stylus velocity in meters per second used in cutting the 400-Hz square wave. The sign ambiguity can be avoided by using a more complicated measurement procedure, but in our experiments the plus sign was always valid, and the extra complication was not warranted.

The effective vertical angles θ_R of the test records were determined by playing each record both forwards and in the reverse direction, taking care that the pickups were properly oriented during reverse playing to duplicate the pickup–groove geometry of normal forward playing. If E_+ denotes the value of voltage E during forward playing, and E_- denotes the value for reverse playing, then it can be shown from tracking theory [8] that the effective vertical recording angle is given by the formula

$$\theta_R = \sin^{-1}[(E_- - E_+) 87.3 \times 10^{-6} R/(kV_{SW})].$$

Our test records (lacquers) were recorded with four bands, each band having a different peak vertical velocity V_{SW}. Each record was used to measure the vertical angles of from six to eleven different pickups in the following way. A pickup was chosen and used to play the four bands forward and in reverse. The value of the effective recording angle θ_R was computed for each band, and the mean angle averaged over the four bands was then computed. This procedure was repeated from six to ten times with a variety of pickups, then all the mean values of θ_R were averaged to yield the grand mean $\overline{\theta}_R$, their standard deviation was computed, and so was the standard error of the mean to verify that the mean was precise. Then the data on forward playing were used to compute the pickup angles θ_P for each band and each pickup using formula (1) with $\theta_R = \overline{\theta}_R$. For each playing of a record this yielded four values of vertical tracking angle θ_P, one for each band. These four values were used to compute the following statistics: the mean $\overline{\theta}_P$, which was our best estimate of the pickup vertical tracking angle; the standard deviation S_{θ_P}, and the standard error of the mean $S_{\theta_P}/\sqrt{4}$.

The results show that data obtained in this way are precise. For example, the vertical angles of four pickups were measured by this method in two independent experiments using two different test records that were cut two months apart. The average difference in measured vertical tracking angles between the two sets of measurements was 1.16 degrees (3.8% of the mean tracking angle), which is within the estimated calibration tolerance of ±5% for the measurement equipment and the square-wave velocity levels cut on the records. (We calibrated these velocity levels by measuring the peak velocity V_P of the 400-Hz fundamental component of the square-wave modulation and then assuming that the peak square-wave velocity was $\pi V_P/4$, which is in accord with the Fourier-series representation of a square wave.)

Another measure of precision is the range of values we obtained for the standard error of the mean tracking angle ($S_{\theta P}/2$). From 27 experiments with ten pickups, using three different test records, we obtained 27 values for $S_{\theta_P}/2$. The largest standard error was $S_{\theta_P}/2 = 0.41$ degree, the smallest was 0.07 degree, and the mean was 0.20 degree. These correspond to standard deviations of 0.82, 0.14, and 0.40 degree.

2. COMPARISON OF THE THREE FM METHODS

So far we have described three FM methods for measuring vertical tracking angles. The experimental procedures and methods of data analysis have been described for each, and standard errors have been cited, which establish that each method yields precise values for the tracking angles of a variety of pickups.

An important subject that has not yet been discussed is the accuracy of these methods. Unfortunately, there are no accepted rigorous methods for *directly* measuring the vertical tracking angles of commercial pickups while they are reproducing audio signals. Therefore indirect methods must be used that permit the tracking angles to be deduced from data on other quantities that can be accurately measured, such as FM deviations. We have four major reasons for believing that FM techniques offer an accurate means for deducing tracking angles: 1) there exist rigorous formulas for computing tracking angles from FM deviation data; 2) tracing distortion and amplitude-modulation distortion due to other nonlinearities in the record–reproduce chain do not cause bias in the data obtained from properly managed FM experiments; 3) the assumptions on which FM methods are based appear to be physically reasonable and not merely convenient; and 4) we used the three FM methods described in this paper to measure the vertical angles of ten pickups in a total of 53 experiments, and the three methods gave results consistent with each other to within the calibration tolerance of the test records and measurement equipment, which is estimated to be ±5%.

Table 1 shows data for comparing typical results obtained with the three measurement techniques. Each row contains data on a single pickup. The first column designates the pickup, and the second column give the average of the three values of vertical tracking angle θ_P measured by the three methods; this average is called the grand mean θ_P. In each case the tracking force was 10 millinewtons (1 gram), and no side thrust compensation was used. The pickups are arranged in order of increasing angle, from 21.80 to 33.00 degrees. The last rows contain the mean of these angles ($\bar{\theta}_P = 28.75$ degrees) and their standard deviation ($S_{\theta_P} = 3.75$ degrees).

The remaining columns contain two statistics on the individual values of tracking angle that were obtained with the three methods. The first statistic is D, which is the difference between the estimated tracking angle obtained with one measurement technique and the grand mean θ_P for all three methods. For example, with pickup 1 the total sine-wave method gave an angle that was 1.60 degrees less than the grand mean of 21.80 degrees.

The second statistic is SE, which is the standard error of the estimated tracking angle obtained with one measurement technique. The standard errors for the total sine-wave method have 19 statistical degrees of freedom, because the angle was estimated by using linear regression to compute two parameters with data from 21 bands on a single test record. For the real-sine-wave method, the standard errors have five degrees of freedom, because the angle was estimated as the average of six measurements obtained from one test record by playing three bands twice to yield separate left-channel and right-channel data. Finally, the standard errors for the total-square-wave method have three degrees of freedom, because the tracking angle was estimated as the average of four measurements obtained by playing the four bands on one test record.

The bottom rows contain statistics for each of the three methods: the mean departure $\bar{D}$ from the grand mean θ_P; the standard deviation S_D of the departures; the mean standard error $\overline{SE}$ of the estimated θ_P; and the mean standard deviation $\bar{S}$ of the data from which the estimated θ_P were computed. The mean departures $\bar{D}$ for the three methods measure the differences in calibration of the three test records and the measuring equipment, and they imply that the three methods would be in perfect agreement on the average if 1.30 degrees were added to the vertical angles deduced from the total-sine-wave method, 0.37 degree were added to the angles deduced from the real-sine-wave method, and 1.47 degrees were subtracted from the vertical angles deduced from the total-square-wave method. There would then remain only the zero-mean scatter about the grand mean θ_P, which is measured by the standard deviations S_D that range from 0.27 to 0.33 degree for the three methods. It is instructive to compare the mean departures with the ten-pickup grand mean ($\bar{\theta}_P = 28.75$ degrees). For the total-sine-wave method $\bar{D} = -1.30$ degrees, which is 4.5% of 28.75 degrees, while the total-square-wave method yielded $\bar{D} = 1.47$ degrees, which is 5.1% of 28.75 degrees. The difference between the $\bar{D}$ for the two sine-wave methods is 0.91 degree (3.2% of 28.75 degrees). These comparisons lead us to the conclusion that the three FM methods give tracking-angle values that are consistent with each other to within the calibration tolerance of the test records and measuring equipment, which is estimated to be ±5%.

The average precision of the estimated θ_P obtained from one measurement method is measured by its mean standard error $\overline{SE}$; and the average variability of the data from which the individual θ_P were computed is measured by the mean standard deviation $\bar{S}$. (No value is given for $\bar{S}$ in the total-sine-wave method because only one linear-regression analysis was performed to estimate each θ_P with this method). Data samples for individual pickups had mean standard deviations of $\bar{S} = 0.22$ degree for the real-sine-wave method and $\bar{S} = 0.32$ degree for the total-square-wave method. The mean standard errors of the estimated vertical angles ranged from $\overline{SE} = 0.09$ degree for the real-sine-wave method to $\overline{SE} = 0.18$ degree for the total-sine-wave method. These statistics indicate that the data obtained for individual pickups with each of the three methods had high precision by engineering standards. This same level of precision was not usually obtained, however, when the vertical angle of a single pickup was measured several times in experiments spaced weeks apart using a fixed test method. The several values of estimated vertical angle obtained from such experiments would typically have a standard deviation in the range of 0.5 to 1 degree. Mechanical instabilities of some pickups may account for some of this deviation.

3. TRACKING ANGLES AT 100 R/MIN (1200 HZ)

The two sine-wave methods both yield values for the tracking angle at 400 Hz. This raises the question as to

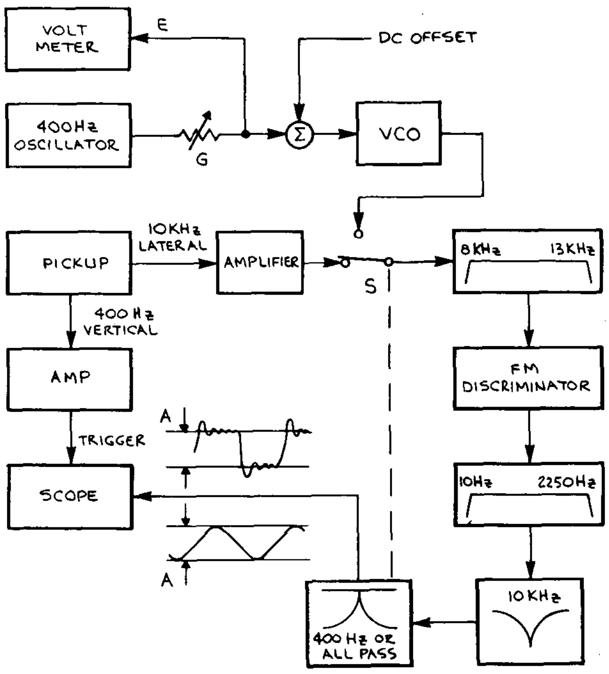

Fig. 4. Block diagram of experimental setup for the total-square-wave method.

whether or not the measured angle is different when the measurement is made at a higher frequency by rotating the test record more rapidly. We conducted one set of experiments to investigate this question. For this purpose we used nine of the pickups covered by Table 1 and remeasured them several weeks later with the total-sine-wave method, except that this time the test record was rotated at a speed of 100 r/min, which caused the recorded 400-Hz tone to be reproduced at 1200 Hz. The resulting values for vertical tracking angle at 1200 Hz (100 r/min) were not significantly different from their 400-Hz values. Specifically, six pickups showed a reduction in angle, while three showed an increase; the mean change was a reduction of 0.21 degree, and the standard deviation of the changes was 0.87 degree; the largest magnitude of change was 1.28 degrees. These changes are similar to the variability we would expect if we were simply to repeat the measurements at 400 Hz.

4. COMPARISON OF FM AND OPTICAL METHODS

In this section the previously described results from FM measurements are compared with vertical-angle data obtained by an optical technique. The vertical tracking angles of the ten pickups in our sample were measured optically by observing the direction of stylus motions under a microscope (equipped with a precision goniometer) while pushing against the stylus with a known slowly varying force. (Similar experiments were described by Woodward [6].) The force applied to the stylus was slowly varied from 8 to 12 mN, and the direction of the force was 15 degrees with respect to the vertical so as to simulate a coefficient of friction in the record groove of 0.26. The ten pickups were measured in two independent experiments by one experimenter. The difference in tracking angles measured in these two optical experiments for each pickup ranged from −1.10 to 1.15 degrees, with a mean difference over the ten pickups of −0.03 degree and a standard deviation of 0.70 degree. The two optically measured angle values for each pickup were averaged to yield an optical estimate of the angle for each pickup. To compare these mean vertical angles obtained by the optical method with the grand means obtained from the three FM methods, the FM grand mean angle for each pickup was subtracted from the optically determined mean angle to obtain a difference angle. These difference angles ranged from −2.66 to 1.80 degrees, with a mean of −0.14 degree and a standard deviation of 1.50 degrees. From these statistics we expect that the optical and 400-Hz FM estimates of vertical angle will agree with each other to within 3 degrees about 95% of the time for the types of pickups represented by our sample. The statistics also yield a 90% confidence interval for the mean difference that extends from −1.0 to 0.72 degree; this indicates that there was no statistically significant average difference between the tracking-angle values obtained by the FM and optical methods.

As a further check on the agreement between the optical and FM methods, the pickups were rank ordered from 1 to

Table 1. Comparison of three FM methods.

Pick-up	Grand Mean θ_P from Three FM Methods (degrees)	Mean Departures and Their Standard Deviations (degrees)		
		Total Sine Wave (degrees)	Real Sine Wave (degrees)	Total Square Wave (degrees)
1	21.80	D = −1.60 SE = 0.13	No data	D = 1.60 SE = 0.17
2	23.43	D = −1.27 SE = 0.16	D = −0.27 SE = 0.08	D = 1.54 SE = 0.11
3	27.38	D = −1.55 SE = 0.14	No data	D = 1.55 SE = 0.24
4	28.12	D = −1.44 SE = 0.13	D = −0.09 SE = 0.08	D = 1.59 SE = 0.20
5	28.60	D = −1.72 SE = 0.28	D = −0.13 SE = 0.03	D = 1.85 SE = 0.16
6	30.01	D = −0.74 SE = 0.22	D = −0.73 SE = 0.14	D = 1.46 SE = 0.17
7	30.65	D = −1.29 SE = 0.17	D = −0.14 SE = 0.07	D = 1.44 SE = 0.28
8	31.85	No data	D = −0.80 SE = 0.06	D = 0.80 SE = 0.08
9	32.62	D = −1.19 SE = 0.17	D = −0.37 SE = 0.14	D = 1.56 SE = 0.13
10	33.00	D = −0.86 SE = 0.21	D = −0.40 SE = 0.15	D = 1.25 SE = 0.07
All ten pickups	$\bar{\phi}_P$ = 28.75 S_{ϕ_P} = 3.75	$\bar{D}$ = −1.30 S_D = 0.33 $\overline{SE}$ = 0.18 ($\bar{S}$ not applicable)	$\bar{D}$ = −0.37 S_D = 0.27 $\overline{SE}$ = 0.09 $\bar{S}$ = 0.22	$\bar{D}$ = 1.47 S_D = 0.28 $\overline{SE}$ = 0.16 $\bar{S}$ = 0.32

D—estimated θ_P minus grand mean θ_P; SE—standard error of estimated θ_P.

10 in order of increasing tracking angle using both the optical and FM data. Then the Spearman rank correlation coefficient [11] was computed to determine the magnitude of agreement between the two sets of rank orders. The coefficient turned out to be 96% (out of a possible 100% for complete agreement), which shows a high degree of correlation.

These results may be summarized as follows.

1) There is no evidence of a statistically significant average difference between tracking angles obtained by the optical and FM methods for the pickups represented by our sample.

2) Certain pickups yield differences between vertical angles measured by the optical and FM methods that cannot be attributed to random experimental error. With the ten pickups in our study the largest of these differences was −2.66 degrees and the standard deviation of all ten differences was 1.50 degrees around a mean of −0.14 degree. Thus for pickups represented by our sample, about 95% are expected to yield optical and FM estimates of the vertical angle that agree with each other to within 3 degrees.

3) The optical method is useful as a laboratory method for investigating vertical angles, but it suffers from the following disadvantages: a) expensive precision microscopic equipment must be used; b) significant time is required by a highly skilled experimenter; and c) the resulting angles, which are measured at a frequency of 0 Hz, do not necessarily agree with the tracking angle at 400 Hz that is effective while playing test records.

5. APPLICATIONS

The total-sine-wave method requires less complex hardware than the other two FM methods. However, many bands on the RCA 12-5-78 test record must be measured, and then a linear regression on the data should be performed to obtain precise estimates of the vertical angle with this method. In contrast, with the real-sine-wave method only a single band on the CBS STR-112 test record must be played, followed by numerical evaluation of a single formula, to obtain an estimate of comparable precision. (For the same precision with our implementation of the total-square-wave method, we estimate that two bands would have to be played.)

Therefore, the real-sine-wave method appears to be the best method of the three for quality control in production-line testing, where sophisticated test equipment can be justified when it reduces the complexity and duration of the test. In principle, a dedicated piece of hardware can be constructed that permits the real-sine-wave method to be carried out in "real time" without off-line computation. Such hardware would permit the vertical angle of a pickup to be read from a meter while a single band on the test record was being played. We plan to describe such a device in a future paper.[1]

6. SUMMARY

The principal contributions of this paper are listed below.

1) A new FM distortion method for measuring vertical tracking angles of phono pickups is described, termed the total-square-wave method. The data obtained by this method are free of bias due to tracing distortion, and a test record with only one band of grooves is required.

2) Data on the measured vertical angles of ten pickups, obtained by three FM-distortion methods, are compared. The three methods are shown to give results consistent with each other to within the calibration tolerance of the test records and measurement equipment, which is estimated to be ±5%.

3) Data on the measured vertical angles of nine pickups show no statistically significant difference between the tracking angles at 400 Hz and 1200 Hz when measured by the total-sine-wave method.

4) Vertical-angle values obtained optically show no statistically significant average deviation from angle values obtained by FM methods for the ten pickups in our sample.

5) Data are presented which verify that both the CBS STR-112 and the RCA 12-5-78 commercially available test records can be used to measure vertical angles precisely and consistently. The average difference between vertical angles measured with these two records was 3.2% (0.9 degree) for a set of nine pickups.

6) The real-sine-wave method is suited to production-line testing for quality control because it can be implemented to yield precise measurements of vertical angle in "real time" while a single band on the CBS STR-112 test record is being played.

7. ACKNOWLEDGMENTS

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The biographies of James V. White and Arthur J. Gust were published in the March issue.